



Spin effects

(spin polarized systems, spin-orbit ...)

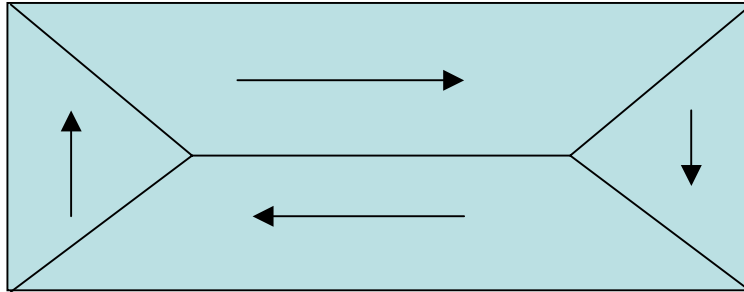
G. Zérah

CEA-DAM Ile de France 91680 Bruyères-le-Châtel

Macroscopic magnetization



- A crystal can be found in different magnetization states.
- The direction of macroscopic magnetization changes inside the crystal
- This can be observed by the magnetic Kerr effect (rotation of polarized light)



- Under the action of an external field, the magnetization $\mathbf{M}(r)$ changes its direction, not its amplitude. $|\mathbf{M}(r)| = M_s$
- Under a very intense magnetic field $\mathbf{M} = M_s \frac{\mathbf{B}}{B}$
- When the magnetic field is turned off, the magnetization distribution is reestablished.
- The domain distribution has an energetic origin (and is not related to defects for instance)

Macroscopic Magnetism



- Magnetic energy
- Landau-Ginzburg like functional

$$E [\mathbf{M}(\mathbf{r})] = E_{an} [\mathbf{M}(\mathbf{r})] + E_{ech} [\nabla \mathbf{M}(\mathbf{r})] + E_{ms} [\mathbf{M}(\mathbf{r})] - \mathbf{M}(\mathbf{r}) \cdot \mathbf{B}_{ext}(\mathbf{r})$$

- The observed magnetization minimizes this functional.
- The first two terms are of microscopic origin
- The third term depends upon the shape of the sample.
- The fourth one describes the interaction with the external field.
- NB: If time is reversed, $M(r) \rightarrow -M(r), B(r) \rightarrow -B(r)$ but the energy is constant

Macroscopic Magnetism



- Different terms in the magnetic energy
- Magnetic anisotropy energy depends upon orientation
- Can exist only if there is a coupling between the lattice and magnetization, through the spin-orbit coupling.

$$\Delta E_{an} [M(r)] \approx 1 \mu eV / atom$$

Macroscopic Magnetism



Anisotropy energy respects the symmetries of the crystal

- Cubic ($K_1 > 0$, [100], $K_1 < 0$ [111] etc..)

$$E_{an} = K_1(m_x^2 m_y^2 + m_x^2 m_z^2 + m_z^2 m_y^2) + K_2 m_x^2 m_y^2 m_z^2 + \dots \quad \vec{m} = \vec{M} / |M|$$

- Hexagonal

$$E_{an} = K_1(m_x^2 + m_y^2) + K_2(m_x^2 + m_y^2) + \frac{K_4}{2} \left((m_x + im_y)^6 + (m_x - im_y)^6 \right) \dots \quad \vec{m} = \vec{M} / |M|$$

Objectives of a magnetic calculation



1. Microscopic determination of the saturation magnetization
 - It might be non uniform inside the crystal cell (non-collinear magnetism)
2. Microscopic determination of the magneto crystalline anisotropy
 - Its most important contribution from the spin-orbit coupling
 - Other an isotropic interactions: spin-spin, spin-other orbits neglected
3. Exchange terms

Some Classes of Magnetic Systems



- Paramagnetism is favored by the presence of bands, which are doubly occupied. But some systems exhibit spontaneous magnetization.
- Collinear ferromagnets :3d Metals : Fe, Co, Ni. Mostly due to spin (orbital moment ~ 0) Stoner criterion: $x_c \times D(E_f)$ large
- Antiferromagnet (anti parallel orientation): Cr, Mn (some phases), ordered alloys: Fe_3Mn ..., insulating compounds MnO , NiS ...: different sublattices with opposite orientations.
- In case the moments do not compensate: ferrimagnetism yields a net magnetic moment. Magnetite: Fe_3O_4 , NiFe_2O_4 Complicated structures: exchange mediated by oxygen
- Helimagnetism: some 4f Rare earth (RKKY interaction), MnAu_2
- More complex non collinear structure: Mn, fcc Fe etc....

Example: Ferromagnetism of 3d metals



- Iron, Nickel and Cobalt: ferromagnets
 - These are 3d elements, for which the atomic 3d WF strongly overlap and form a band of a given width W
 - A qualitative understanding of the existence of magnetism is given by the Stoner criterion.
 - Assume a constant DOS (D), and decompose the energy as band+exchange contributions
 - Correlation effects has a structure: $Un_{\uparrow}n_{\downarrow}$
 - If $DU > 1$, then one spin channel will be filled at the expense of the other.

$$E = E_B + E_{XC} = \int_0^{\varepsilon_{\uparrow}} \varepsilon D(\varepsilon) d\varepsilon + \int_0^{\varepsilon_{\downarrow}} \varepsilon D(\varepsilon) d\varepsilon + Un_{\uparrow}n_{\downarrow}$$

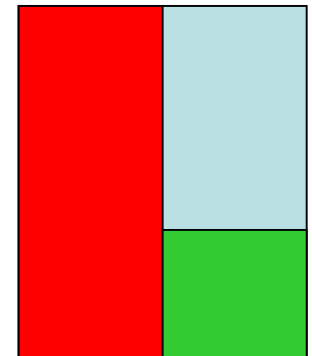
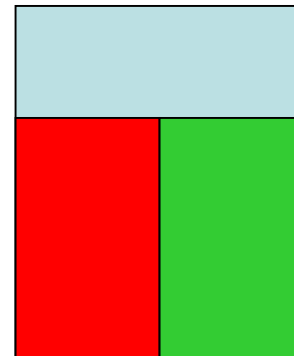
$$= \frac{1}{2D} (n_{\uparrow}^2 + n_{\downarrow}^2) + Un_{\uparrow}n_{\downarrow} = \frac{n^2}{2D} + \left(-\frac{1}{D} + U\right)n_{\uparrow}n_{\downarrow} = \frac{n^2}{2D} + \left(-\frac{1}{D} + U\right) \frac{n^2(1-\zeta^2)}{4}$$

$$D(\varepsilon) = D = Cste$$

$$D\varepsilon_{\uparrow} = n_{\uparrow}$$

$$n_{\uparrow} + n_{\downarrow} = n$$

$$\zeta = \frac{n_{\uparrow} - n_{\downarrow}}{n}$$



Spin DFT



- In principle, the DFT is an exact theory and so there is no need for a particular consideration for spin.
- But, the usual approximations generalize easily if we consider spin explicitly
- One complements the external potential V , with a magnetic field B along z (next for \mathbf{B})
- The system will polarize, and one gets a up and a down electron density: $n_{\uparrow}(\mathbf{r}), n_{\downarrow}(\mathbf{r})$
- The energy of the system will be a functional of the density and the magnetization (supposed along z).

$$E = K[n_{\uparrow}] + K[n_{\downarrow}] + E_H[n] + E_{xc}[n_{\uparrow}, n_{\downarrow}] \\ + \int d\mathbf{r} n(\mathbf{r}) V_{ext}(\mathbf{r}) - \int d\mathbf{r} m(\mathbf{r}) B_{ext}(\mathbf{r})$$

$$n(\mathbf{r}) = n_{\uparrow}(\mathbf{r}) + n_{\downarrow}(\mathbf{r})$$

$$m(\mathbf{r}) = n_{\uparrow}(\mathbf{r}) - n_{\downarrow}(\mathbf{r}), \text{ in units of } \mu_B$$

The Kohn-Sham equations



- If we minimize the energy, we obtain two equations, one for each spin channel (we set $B_{\text{ext}}=0$)

$$\left\{ -\frac{\Delta}{2} + V_{\text{eff}}^{\sigma}(\mathbf{r}) \right\} \Psi_{\sigma}^i = \varepsilon_{\sigma}^i \Psi_{\sigma}^i \quad \sigma = \uparrow \downarrow$$

$$n^{\sigma}(\mathbf{r}) = \sum_i f(\varepsilon_{\sigma}^i) |\Psi_{\sigma}^i(\mathbf{r})|^2$$

$$V_{\text{eff}}^{\sigma}(\mathbf{r}) = V_{\text{ext}}(\mathbf{r}) + V_h(\mathbf{r}) + V_{xc}^{\sigma}(\mathbf{r})$$

$$V_{xc}^{\sigma}(\mathbf{r}) = \frac{\delta E_{xc}(n_{\downarrow}, n_{\uparrow})}{\delta n_{\sigma}}$$

$$n(\mathbf{r}) = n^{\downarrow}(\mathbf{r}) + n^{\uparrow}(\mathbf{r})$$

$$m(\mathbf{r}) = n^{\uparrow}(\mathbf{r}) - n^{\downarrow}(\mathbf{r})$$

The LSDA



- The total energy is also given by the sum over states:

$$E = \sum_{i,\sigma} \varepsilon_{i,\sigma} - \frac{1}{2} \int d\mathbf{r} n(\mathbf{r}) V_h(\mathbf{r}) - \sum_{\sigma} \int d\mathbf{r} n_{\sigma}(\mathbf{r}) V_{xc}^{\sigma}(\mathbf{r}) + E_{xc} [n_{\uparrow}, n_{\downarrow}]$$

- The exchange correlation energy of the electron gas can be written as:

$$E_{xc}^{LDA} [n_{\uparrow}, n_{\downarrow}] \approx \int n(\mathbf{r}) \varepsilon_{xc}(n_{\uparrow}, n_{\downarrow}) d\mathbf{r}$$

$$\varepsilon_{xc}(n_{\uparrow}, n_{\downarrow}) = \varepsilon_x(n_{\uparrow}, n_{\downarrow}) + \varepsilon_c(n_{\uparrow}, n_{\downarrow})$$

$$\varepsilon_x(n_{\uparrow}, n_{\downarrow}) = -3 \left(\frac{3}{4\pi} \right)^{1/3} \frac{1}{n} (n_{\uparrow}^{4/3} + n_{\downarrow}^{4/3})$$

$$\varepsilon_c(n_{\uparrow}, n_{\downarrow}) \text{ by QMC}$$

$$E_{xc}^{GGA} [n_{\uparrow}, n_{\downarrow}] \approx \int n(\mathbf{r}) \varepsilon_{xc}(n_{\uparrow}, n_{\downarrow}, \nabla n_{\uparrow}, \nabla n_{\downarrow}) d\mathbf{r}$$

Perdew, Burke, Ernzerof PRL, 77, 3865 (1996): PBE

Stoner revisited



- Each spin channel yields a different potential due to the xc term
- To study the stability of the paramagnetic phase, one can expand this potential for a small magnetization term

$$V_{xc}(\mathbf{r}) = V_{xc}^0(\mathbf{r}) \pm m(\mathbf{r})V_{xc}^1(\mathbf{r})$$

$$V_{xc}(\mathbf{r}) = V_{xc}^0(\mathbf{r}) \pm \frac{1}{2}IM \quad \text{If second term } \sim \text{constant: rigid shift}$$

$$M = \int^{\varepsilon_f} \left[D^0\left(\varepsilon + \frac{1}{2}IM\right) - D^0\left(\varepsilon - \frac{1}{2}IM\right) \right] d\varepsilon$$

$$\text{Non zero solution at small } M \Rightarrow ID^0(\varepsilon_f) > 1$$

Stoner revisited



- More rigorously (Janak, 1977): compute the spin susceptibility and the criterion is satisfied when it is negative.
-

$$\chi = \frac{\chi_0}{1 - \chi_0 I}$$

$$I = \int d\mathbf{r} \gamma^2(\mathbf{r}) |K(\mathbf{r})|$$

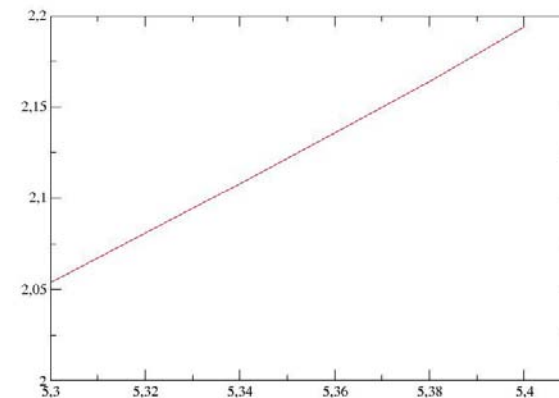
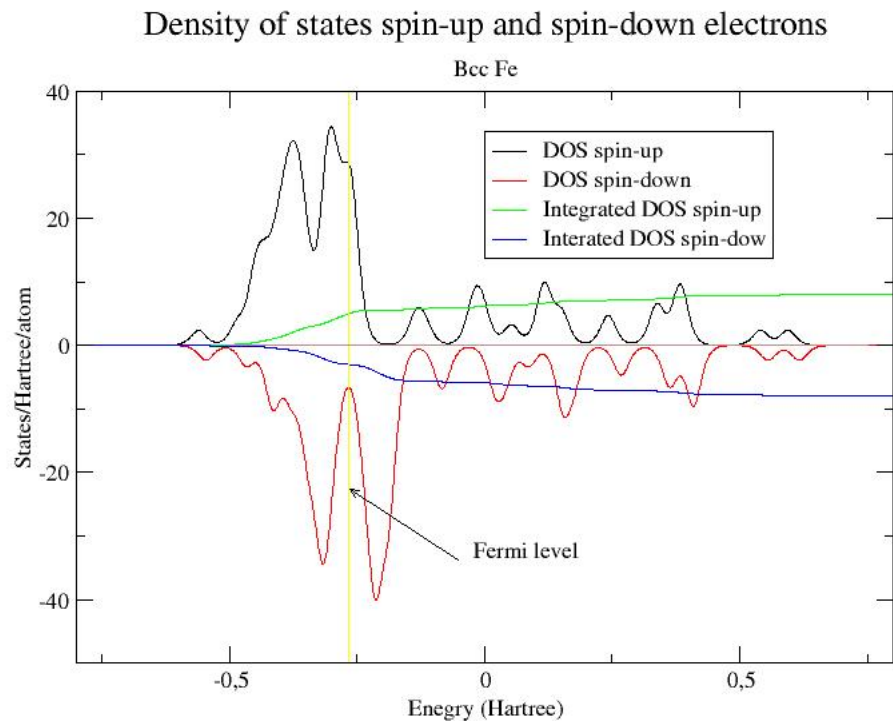
$$D^0(\varepsilon_f) \gamma(\mathbf{r}) = \frac{d}{d\varepsilon_f} \rho(\mathbf{r})$$

$$\left\{ \delta^2 E_{xc} [\rho, \vec{m}] / \delta m(\mathbf{r}) \delta m(\mathbf{r}') \right\} = 2K(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}')$$

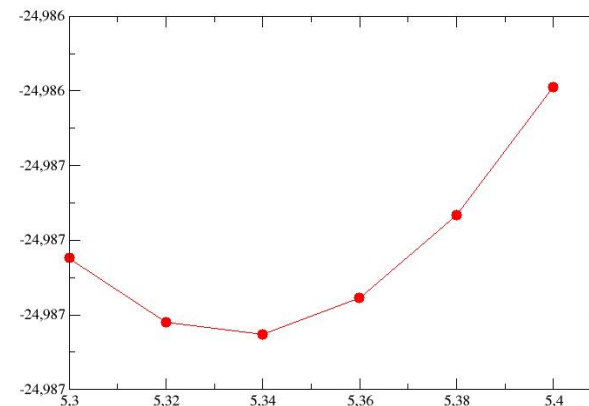
Example of a Pseudopotential calculations (norm conserving)



- BCC Fe ~rigid shift of DOS
- Magnetization strongly dependent of cell parameter



Magnetization as a function of cell (a.u.)



Energy as a function of cell (a.u.)

Calculations using a Pseudo potential



- Iron has been much studied
 - LSDA will predict a correct magnetic moment (2.17 vs 2.22 exp)
 - BUT, the NM fcc structure has a lower energy than the FM bcc
 - This is corrected but using the (PBE) GGA
- Pseudo potentials are generally constructed on non magnetic configurations and used for magnetic systems ...
- Also, the use of non local core corrections is essential (exercice: compare magnetization for bcc Fe, using 26fe.pspnc and 26fe.hgh)
- But, best results are obtained with PAW (cf talks by Jollet Torrent): this is an all electron theory with a psp look (and includes nlcc)

Results of calculations



- 3d ferromagnetic elements (Fe, Co, Ni): very good prediction of the magnetization with GGA
- 3d non ferromagnetic elements (Mn, Cr): complicated spin arrangement (e.g. spiral) non compatible with the hypothese of // spins
- Intermetallic compounds (that is compounds of Cu ,Ni, Co, Fe, Mn, Cr), yields good values of magnetization
- Magnetic Oxydes: classical examples of failure of the LSDA (or GGA)
- This failure is presumably due to a neglect of the on site correlation (U term) of localized orbital.
- LDA includes this only on average (cf the example of Ce)
- This failure is the source of many improvements of LDA, GGA: LDA+U, SIC, Hybrids functionals (HF+LDA), etc...

Non collinear Magnetism



- Up to now, we considered moments all parallel along a given axis
- But there are cases where it is not justified:
 - In some systems, the ground state is non collinear: in Mn, in solid oxygen, in Mn_4N , in fcc Fe etc...
 - Spin excitations (“spin waves”) must take into account non parallel configurations
 - The exchange coupling parameter of the Heisenberg Hamiltonian requires a non collinear arrangement
- This requires a generalization of spin DFT to account for a vector $\mathbf{m}(\mathbf{r})$

Spin 1/2 reminder



- Spin state

$$|s\rangle = a|+\rangle + b|-\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

- Magnetic moment along x

$$S_x = \mu_B \langle s | \sigma_x | s \rangle$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The WF as a two component spinor



- In each point in space, the wave function possesses two spin components, and the KS principle is generalized towards a duality between (density, magnetization) and external fields (potential, magnetic field)

$$\psi(r) = \begin{bmatrix} \psi_{\uparrow}(r) \\ \psi_{\downarrow}(r) \end{bmatrix}$$

- The electronic density can be written:

$$\rho(r) = [\psi_{\uparrow}(r) \quad \psi_{\downarrow}(r)]^* I \begin{bmatrix} \psi_{\uparrow}(r) \\ \psi_{\downarrow}(r) \end{bmatrix} = |\psi_{\uparrow}(r)|^2 + |\psi_{\downarrow}(r)|^2$$

Direction of magnetization in space



- The magnetization along x, for instance:

$$\begin{aligned}M_x(r) &= \begin{bmatrix} \psi_{\uparrow}(r) & \psi_{\downarrow}(r) \end{bmatrix}^* S_x \begin{bmatrix} \psi_{\uparrow}(r) \\ \psi_{\downarrow}(r) \end{bmatrix} \\ &= \mu_B \begin{bmatrix} \psi_{\uparrow}(r) & \psi_{\downarrow}(r) \end{bmatrix}^* \sigma_x \begin{bmatrix} \psi_{\uparrow}(r) \\ \psi_{\downarrow}(r) \end{bmatrix} \\ &= \mu_B \begin{bmatrix} \psi_{\uparrow}(r) & \psi_{\downarrow}(r) \end{bmatrix}^* \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \psi_{\uparrow}(r) \\ \psi_{\downarrow}(r) \end{bmatrix} \\ &= \mu_B (\psi_{\uparrow}(r)^* \psi_{\downarrow}(r) + \psi_{\downarrow}(r)^* \psi_{\uparrow}(r))\end{aligned}$$

Another view: the spin density matrix



- Magnetization

$$\vec{M}(\mathbf{r}) = \begin{bmatrix} \psi_{\uparrow}(\mathbf{r}) & \psi_{\downarrow}(\mathbf{r}) \end{bmatrix}^* \vec{S} \begin{bmatrix} \psi_{\uparrow}(\mathbf{r}) \\ \psi_{\downarrow}(\mathbf{r}) \end{bmatrix}$$

- Spin density matrix

$$\rho^{\alpha\beta}(\mathbf{r}) = \psi_{\alpha}^*(\mathbf{r})\psi_{\beta}(\mathbf{r}) \rightarrow \rho^{\alpha\beta}(\mathbf{r}) = \sum_{kj} f_{kj} \psi_{\alpha}^{*kj}(\mathbf{r})\psi_{\beta}^{kj}(\mathbf{r})$$

Magnetization and spin density matrix



- We can equivalently use each object:

$$\vec{m}(r) = \sum_{\alpha\beta} \rho^{\alpha\beta}(r) \vec{\sigma}_{\alpha\beta} \quad \rho(r) = \sum_{\alpha\beta} \rho^{\alpha\alpha}(r)$$

$$2\rho^{\alpha\beta}(r) = \rho(r)\delta_{\alpha\beta} + \vec{m}(r) \cdot \vec{\sigma}^{*\alpha\beta}$$

- Note the *

Energy functional (GLSDA)



- Energy is now a functional of energy and magnetization vector \mathbf{m}

$$E[\rho^{\alpha\beta}(r)] = T[\rho^{\alpha\beta}(r)] + E_H[\rho(r)] + E_{xc}[\rho^{\alpha\beta}(r)] + \int V_{ext}(r)\rho(r)dr$$

$$T[\rho^{\alpha\beta}(r)] = -\frac{\hbar^2}{2m} \sum_{jk} \int f_{kj}(\psi_{kj}^{*\alpha}(r)\Delta\psi_{kj}^{*\alpha}(r) + \psi_{kj}^{*\beta}(r)\Delta\psi_{kj}^{*\beta}(r))dr$$

$$E_{xc}[\rho^{\alpha\beta}(r)] = E_{xc}[\rho(r), \mathbf{m}(r)]$$

- One can equivalently use the spin density matrix.
- The variational principle applied to it leads to the new Hamiltonian
- As usual, one needs to construct the exchange correlation

The « local spin density approximation »(LSDA)



- One extends the usual (LDA) exchange correlation term to include the presence of vector magnetization

$$E_{xc} = \int \rho(r) \varepsilon_{xc}(\rho(r), |\mathbf{m}(r)|) dr = \int \rho(r) \varepsilon_{xc}(\rho^{\alpha\beta}(r)) dr$$

- If one uses the electron gas as a reference, the magnetization direction is immaterial (note, this is easy to express with the help of the magnetization vector)
- We can use the same formula for the LDA as the usual collinear magnetic case, since it only depends on $|\mathbf{m}|$
- In principle, the GGA should include the rotationally invariant parts of $\nabla\mathbf{m}$, but is not yet done...



$$H^{\alpha\beta} = -\frac{1}{2}\Delta\delta_{\alpha\beta} + V_{eff}^{\alpha\beta} = \frac{\delta E}{\delta\rho^{\alpha\beta}(r)}$$

$$V_{eff}^{\alpha\beta} = V_H(\rho(r))\delta_{\alpha\beta} + V_{xc}^{\alpha\beta}(\rho^{\alpha\beta}(r)) + V_{ext}(r)$$

Kohn-Sham Hamiltonian (contd)



$$V_{xc}^{\alpha\beta}(\rho^{\alpha\beta}(r)) = \frac{\delta E_{xc}(\rho^{\alpha\beta}(r))}{\delta \rho^{\alpha\beta}(r)}$$

$$V_{xc}^{\alpha\beta}(\rho^{\alpha\beta}(r)) = V_{xc}(\rho^{\alpha\beta}(r))\delta_{\alpha\beta} - \mu_B \vec{b}(\rho^{\alpha\beta}(r)) \cdot \vec{\sigma}^{\alpha\beta}$$

$$\vec{b}(\rho^{\alpha\beta}(r)) = -\frac{\delta E_{xc}(\rho^{\alpha\beta}(r))}{\delta \mathbf{m}(r)} = \hat{\mathbf{m}}(r)\rho(r) \frac{\partial \varepsilon_{xc}(\rho^{\alpha\beta}(r))}{\partial |m(r)|}$$

Energy functional

Expression with the help of eigenvalues



$$E[\rho^{\alpha\beta}(\mathbf{r})] = \sum_{\mathbf{k}j} f(\varepsilon_{\mathbf{k}j}) \varepsilon_{\mathbf{k}j} - \int V_{eff}(\mathbf{r}) \rho(\mathbf{r}) d\mathbf{r} + \int \mathbf{B}_{eff}(\mathbf{r}) \mathbf{m}(\mathbf{r}) d\mathbf{r} \\ + \int V_{ext}(\mathbf{r}) \rho(\mathbf{r}) d\mathbf{r} + E_H[\rho(\mathbf{r})] + E_{xc}[\rho(\mathbf{r}), \mathbf{m}(\mathbf{r})]$$

Spin-orbit: The Dirac Hamiltonian

- Dirac's equation (ϕ and ψ two components spinors)

$$\left[E + 2mc^2 - V \right] \phi + c\sigma \cdot p\psi = 0$$

$$\left[E - V \right] \psi + c\sigma \cdot p\phi = 0$$

- Low E: scalar relativistic approximation + spin-orbit coupling

$$\phi \approx (v/c)\psi \quad \text{ordre } (1/c)^2 = (1/137)^2$$

$$\frac{p^2}{2} F + V(r)\phi$$

$$- \frac{p^4}{8c^2} F - \frac{1}{4c^2} \nabla V(r) \cdot \nabla \phi$$

$$+ \frac{1}{4c^2} \sigma \cdot (\nabla V(r) \times p\phi) = \varepsilon \phi$$

Spin-orbit coupling



- Spherical potential

$$\nabla V (r) = \frac{1}{r} \frac{dV}{dr} \mathbf{r}$$

$$\frac{1}{4 c^2} \boldsymbol{\sigma} \cdot (\nabla V (r) \times \mathbf{p}) = \frac{1}{r} \frac{dV}{dr} \boldsymbol{\sigma} \cdot (\mathbf{r} \times \mathbf{p}) = \frac{1}{r} \frac{dV}{dr} \boldsymbol{\sigma} \cdot \mathbf{l}$$

- Order of magnitude for a valence orbital

$$a \approx 1 / Z , V \approx Z^2$$

$$\langle \psi | r^{-1} dV / dr | \psi \rangle \approx (Z / c)^2$$

- -> Much better for the valence pseudo orbital $\approx (Z_{\text{eff}} / c)^2$

Spin orbit projector



- The **L.S** operator
 - The angular momentum operator has invariant spaces indexed by l and of dimension $(2l+1)$. A projector on this space is denoted

$$|l \rangle \langle l|$$

- The same is true for spin (with $l=1/2$)

$$|s \rangle \langle s|$$

- A projector on spin and angular momentum space is

$$|ls \rangle \langle ls|$$

The L.S operator



- On this space the $L+S=J$ operator has two invariant subspaces
 - Angular momentum $l+1/2$ and $l-1/2$ of dimensions $2l+2$ and $2l$
 - Now, $J^2 = (L + S)^2 = L^2 + S^2 + 2L.S$
 - So we derive the meaning of

$$|\mathbf{L.S}ls \rangle \langle ls|$$

- Project on the ls space, and apply the spin-orbit coupling

How to build Pseudo potentials



- For a potential with a spherical symmetry, the solutions of Dirac's equation have the form:

$$\phi_{l+1/2}(r) \begin{pmatrix} \left(\frac{l+m+1}{2l+1}\right)^{1/2} Y_l^m \\ \left(\frac{l-m}{2l+1}\right)^{1/2} Y_l^{m+1} \end{pmatrix} \quad \phi_{l-1/2}(r) \begin{pmatrix} \left(\frac{l-m+1}{2l+1}\right)^{1/2} Y_l^{m-1} \\ -\left(\frac{l+m}{2l+1}\right)^{1/2} Y_l^m \end{pmatrix}$$

- One can build a pseudo potential for which the wave functions have the same shape beyond a certain radius, by only adding a spin orbit term to it

$$V_{NL} = \sum_l \left(\sum_m V_l^{SR}(r, r') |ls\rangle \langle ls| + \sum_m V_l^{SO}(r, r') \mathbf{L} \cdot \mathbf{S} |ls\rangle \langle ls| \right)$$

$$V_l^{SR(O)}(r, r') = \sum_{ij} \alpha_{ij} P_i(r) P_j(r')$$

How to use a plane wave basis



- Each component is developed on a basis
- For instance, in plane waves, the basis is

$$G \uparrow = \begin{bmatrix} e^{2i\pi G \cdot r} \\ 0 \end{bmatrix} \quad \text{et} \quad G \downarrow = \begin{bmatrix} 0 \\ e^{2i\pi G \cdot r} \end{bmatrix}$$

- The components of the wave function are:

$$\psi(\mathbf{r}) = e^{i\mathbf{k} \cdot \mathbf{r}} \begin{bmatrix} \psi_{\uparrow}(\mathbf{r}) \\ \psi_{\downarrow}(\mathbf{r}) \end{bmatrix} = e^{i\mathbf{k} \cdot \mathbf{r}} \begin{bmatrix} \sum_G a_{G\uparrow} e^{i\mathbf{G} \cdot \mathbf{r}} \\ \sum_G a_{G\downarrow} e^{i\mathbf{G} \cdot \mathbf{r}} \end{bmatrix}$$

- That is two times more coefficients, and matrices 4 times bigger

Expression in the plane wave basis

- The important term is $\langle G|V_{NL}|G'\rangle$



$$\langle G|Q_i(r)Q_j(r')|ls\rangle\langle ls|G'\rangle = 4\pi(2l+1)f_i(|G|)f_i(|G'|)P_l(\hat{G}\cdot\hat{G}')$$

$$\langle Gs_1|Q_i(r)Q_j(r')\mathbf{L}\mathbf{S}|ls\rangle\langle ls|G's_2\rangle = -i4\pi(2l+1)f_i(|G|)f_i(|G'|)P_l(\hat{G}\cdot\hat{G}')\langle s_1|\mathbf{S}|s_2\rangle\cdot\mathbf{G}\times\mathbf{G}'$$

$$f_i(|G|) = 4\pi\int_0^{+\infty} Q_i(r)\exp(iGr)r^2 dr$$

- See Gonze et al. Comp. Mat. Sci., 25 (2002)

Effect of spin-orbit coupling upon total energy



- Heavy metals
 - « norm conserving » pseudo potentials
 - Experimental equilibrium phases
 - N. Richard et S. Bernard(PRB,66,2002)

	GGA	GGA+SO	EXP	GGA	GGA+SO	EXP
U	145GPa	132Gpa	135Gpa	19.91	20.33	20.56
Np	180Gpa	165Gpa	120Gpa	18,20	19.83	19.23

Conclusions



- Magnetic moments generally well reproduced by GLSDA
- But magnetic oxides exhibit localized states which are not well described by LDA (Mott insulators e.g. due to correlations are described as metals)
- Anisotropy is well described for “hard” magnets, less well for soft magnets
- Temperature effects on K are nearly untouched
- Spin-orbit coupling has a smaller effect on total energy in a psp context than in all electrons calculations
- But changes the DOS substantially for heavy metals

Bibliography



- General:
 - R. Martin's Book
- Magnetism
 - Introductory (with emphasis on materials and properties)
 - Magnetic Material, Nicola Spaldin, Cambridge (2003)
 - Advanced
 - Magnetism and the electronic structure of crystals, Gubanov et al., Springer 1992